Bi-level Optimisation of Differential Pricing Strategies for Oversaturated Metro Systems

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Abstract
With growing concerns about traffic demand management practices in the large-scale metro networks, there is consensus that differential pricing performs effectively in dealing with crowding during commuting peaks. Instead of taking mandatory measures to suspend passengers flooding onto the platforms, the superiority of differential pricing lies in incentivising commuters to shift to peak-avoidance choice spontaneously. To provide both practical and theoretical implications, a comprehensive framework for developing differential pricing strategies is presented in this work. Specifically, a stated preference survey regarding commuters’ response to a newly implemented strategy is conducted in support of modelling departure time and mode shift joint choice behaviour. The proposed bi-level programming model specifies a multi-objective optimisation model in the upper level and a nested logit based stochastic user equilibrium model in the lower level. Based on an empirical case in Beijing, off-peak discount, extra peak charge and both above strategies are thoroughly evaluated in terms of metro ridership variations, mode share patterns, ticket revenue and generalised travel cost of commuters. The results of price sensitivity analysis highlight the universal features and application conditions of different strategies from a practical point of view. Furthermore, six optimal differential pricing strategies tailored by typical decision-making priorities are successfully developed, with the optimal off-peak discount of up to 70% and extra peak charge working best at 60%.

Keywords dynamic pricing · multi-objective optimisation · commuting behaviour · nested logit · underground

1 Introduction
Owning to the massive capacity, high efficiency and low energy consumption, urban rail transit has been widely perceived as an advocated mode choice for the public and has played a crucial role in alleviating traffic congestion and promoting sustainable mobility in many megacities. As the most reliable way for daily commuting, continuous expansion of the metro network is attracting a huge growth in the number of commuters during peak hours and has led to the emergence of oversaturated metro systems throughout the world (Liu et al. 2018).

As a consequence, an imbalance between transport supply and travel demand inevitably arises. Taking Beijing metro as an example, there are 23 lines and 394 stations in operation, with a total mileage of 678 kilometres by the end of 2019, ranking the system as second largest in the world. On average, approximately five million passengers use the metro network during the morning peak from 7:00 am to 10:00 am. The broad application of automated fare collected (AFC) system provides access to data that allows the ridership patterns to be analysed from a statistical perspective. Intuitively, Figure 1 presents the hourly ridership variations of the whole network on a single weekday in 2019.
As can be seen in Figure 1, hourly ridership distributes unevenly throughout the day, with two distinct peaks that are most probably led by commuting activities. To highlight the mismatch between demand and supply, Figure 2 intends to give an interpretation of the crowdedness degree of Line 4 in terms of the section load rate. For each section between two stations, the load rate represents the ratio of actual passenger loads to the maximum train capacity. The reason for choosing Line 4 lies in its ideal location that connects north and south parts of Beijing, which falls right into the scope of the main commuting corridors.

From the section load rates in both upward and downward directions presented in Figure 2, it is evident that the rush hours in the two directions are totally different in both space and time. In fact, the upward direction departs from the suburb to the city centre area, which leads to a peak of load rates among the first few sections during the morning peak. Oppositely, the bottleneck of downward direction occurs in sections located in the centre area during evening rush hours. In general, when the localised load rate rises higher than 100%, equivalent to nine people standing per square metre, platforms and carriages can no longer meet the comfort standard for passengers. Therefore, to avoid further loss of passenger satisfaction and potential risks in train operations, solutions at both the supply and demand side have been proposed, as outlined below.

In regards to the supply-side, the enhancement of transport capacity takes supply to the next level physically by compressing train departure intervals or introducing new lines. Research on the optimisation of train scheduling has received growing attention in the past few years (Qi et al. 2018; Wang et al. 2018; Yin et al. 2017). These studies have offered valuable implications on developing train schedules with higher efficiency and a lower budget, but only metro systems with growth potential can benefit from this. The application performance on well-established metro systems is unsatisfactory, especially for those already operating in an oversaturated status. Indeed, for many of the most crowded lines, the train departure interval has already been reduced to the shortest possible, leaving limited room for further improvement subject to safety consideration and existing signal equipment. Similarly, the development of new lines in highly developed urban environments has significant
planning and financial implications.

Another category of solutions attempts to make efforts from the demand-side, which intend to coordinate the demand to meet with existing supply. This belongs to the domain of travel demand management (TDM), which refers to various strategies to alleviate the effects of recurrent congestion by re-distributing travel demand in space and time (Raby 2014). Experience from long-term practice has demonstrated its effectiveness in both mandatory and incentivised ways. More specifically, mandatory measures stand for flow control strategies. For example, to avoid crowd gathering at the platform during peak period, batch release is usually taken at the security point, e-gate, or even transfer channel. By slowing down the movement of passenger flow within the station, the congestion on platforms and carriages can be relieved effectively. In this respect, researchers keep making efforts to develop the optimal flow control strategy for oversaturated metro systems (Jiang et al. 2017; Yang et al. 2017; Shi et al. 2019). However, mandatory measures inevitably have negative impacts on passengers’ travel experience and probably lead to a loss of ridership due to the worse service quality. On the other hand, incentivisation measures allow commuters to avoid peak travel voluntarily. For example, differential pricing is commonly recognised as a practical way for incentivising possible adjustment of unnecessary or flexible travel demand, which works as an economic tool and takes effects on the individual decision-making process. In this regard, mandatory measures should at best be seen as supplementary to incentivisation measures so as to achieve sustainable development. There is thus a need to better understand the impacts of pricing on commuter’s travel behaviour, and this forms the key motivation of the present paper. Moreover, this study presents a bi-level optimisation framework for developing differential pricing strategies from the perspective of balancing spatiotemporal mismatches between supply and demand, along with an empirical case study in Beijing, China.

2 Literature Review

Differential pricing, also known as demand pricing or time-based pricing, is a pricing strategy that allows flexible prices for products or services determined by current market demands. Although there is a wide range of industries where differential pricing has been successfully applied, including retailing, energy, tourism and public transport, it is important to note that differential pricing only takes effect when the target market has elastic demand. The commuting demand for metro shows elasticity for either adjusting regular departure time or even shifting to other travel modes. Therefore, measuring the elasticity of travel demand plays a crucial role in developing appropriate differential pricing strategies, and further progress on incorporating elastic demand into the policy-making process is needed. Based on the above considerations, an overview of real-world differential pricing cases is presented for practical reference. Subsequently, studies on commuting behaviour and travel demand analysis as well as existing methods for optimising differential pricing strategies, are reviewed as follows.

2.1 Practical Cases of Differential Pricing in Metro Systems

Differential pricing strategies are currently used in metro systems of many megacities over the world. In Singapore, Travel Smart Programme (TSP) was first launched in 2012, with the purpose of encouraging commuters to move their departure time outside of peak hours and earning points or rewards in return. To further incentivise off-peak travel, Free Pre-Peak Travel (FPPT) was implemented in 2013. Afterwards, the Public Transport Council (PTC) announced an updated pre-peak discount scheme to replace the current free-of-charge policy in December 2017. Commuters who tap in before 7:45 am can enjoy up to 50 cents of discount in the one-way trip, taking about a third of the full price.

London underground is another typical case that has put the differential pricing strategy in place for years. The off-peak fares are available for commuters selecting either Oyster card or contactless card payment, when travelling outside 6:30 am - 9:30 am and 4:00 pm - 7:00 pm on weekdays. The amount of discount depends on the distance travelled. The highest discount applies to travels to Zone 6, and the amount is up to 2 pounds as nearly two-fifths of the full price.

In New York City, the Metropolitan Transportation Authority (MTA) undertakes the operations of the subway, bus and Staten Island railway. Although no discount is offered during off-peak times, reduced fares specific to elderly or those with qualifying disabilities are available for express bus except during weekday rush hours. The
limited usage of Reduced-Fare Metrocard intends to achieve similar goals as differential pricing.

Tokyo Metro provides commuters with an economical product called Commuter Railway Pass. It requires commuters to specify two stations near their home and workplace in advance of allowing unlimited rides on the specified route, with alternatives of a one, three or six-month plan. Although no off-peak tickets are offered for local commuters, 12 packs of Off-peak Coupon Tickets are sold, mainly for tourists, for travel between 10:00 am - 4:00 pm. The price of a set of coupon tickets is equal to that of 10 regular tickets.

In Beijing, a differential pricing scheme was first introduced in December 2015. During the first year trial, passengers who tap in at any station of specific lines before 7:00 am automatically enjoy 30% off for their whole trip. The reduced fare would be deducted from their e-card when tapping out at any station. The off-peak discount remains valid if there are any other sale events for e-card holders. Soon afterwards, the authority raised the off-peak discount to 50% for more potent incentivising effects. Despite this, daily metro ridership kept growing dramatically and surpassed eleven million in 2019. Disputes on future implementation plans of differential pricing remain ongoing so far. The oversaturated system is gradually becoming a common situation that many megacities face. In order to develop effective strategies and get over experience-based decision making, contributions on quantitative analysis and scientific optimising methods are both in urgent needs.

2.2 Empirical Effects Analysis of Differential Pricing

Travel demand analysis is one of the most traditional topics in the traffic planning domain that provides essential references for the operations management of public transport. To highlight the effects of differential pricing, only work looking at commuting activities or travel demand analysis during peak hours is reviewed in this part.

Bianchi et al. (1998) investigated the performance of the newly started time-of-day pricing strategy in the Santiago Metro. To investigate the behavioural heuristics hidden behind that, the SP survey was conducted to provide insights on change in travel time, price difference and comfort. Both ordinal probit and binary logit models showed high validity against the observed reality, demonstrating commuters’ willingness to adjusting departure time when the off-peak discount or improved comfort is accessible in return.

Ozbay et al. (2006) evaluated the traffic impacts of the time-of-day tolling program launched in New York and New Jersey. With the traffic data routinely collected on toll lanes, a statistically significant shift from peak to off-peak was confirmed. However, the new tolling program slightly affected the traffic status on weekends.

Currie (2010) presented a quantitative analysis of how early bird ticket affects rail ridership in Melbourne. There were around 8,000 to 9,000 passengers who used that ticket every single weekday, 23% of them shifted their departure time with an average of 42 minutes earlier than usual. It should be mentioned that not all passengers who used that ticket contributed to the demand shift to off-peak, as some of those previously departed early are naturally eligible for the new benefit without the need of changing regular plans. Although that program brought $6 million loss in ticket revenue, it was viewed as a powerful solution to rail overcrowding in that time.

On the issue of behaviour adjustment in the context of pricing changes, Song (2011) and Wang (2011) investigated the public’s general opinions on conventional pricing and differential pricing problems. Their survey focused more on the socio-demographic characteristics, travel habits and acceptable price ranges of metro commuters in Hangzhou and Guangzhou of China, respectively. However, the hypothetical scenarios in the above studies are oversimplified by only presenting delay time and ticket price to the respondents, thereby ignoring the possibility of adjusting departure time.

Ben-Elia and Ettema (2011) conducted a 13-week field survey to observe rush-hour avoidance intentions after the introduction in the Netherlands of the Spitsmijden project, which set rewards on different levels and types - either monetary or in-kind for driving commuters. With the aid of state-of-the-art detection equipment, participants’ itineraries were tracked and modelled by a discrete mixed model. The results revealed that rewards promote shifts to off-peak period, public transport and working from home. The work found that significant influencing factors included work time flexibility, habitual features, availability of travel information and even weather conditions.

Peer S et al. (2015) targeted annual train pass holder in the Netherlands to explore trip scheduling preferences in a peak avoidance experiment. A customised application was installed on participants’ smartphones in advance to record their commuting activities. Collected data indicated that rewards motivated a 22% fall in the number of peak trips. Multinomial Logit (MNL) model estimation results emphasised the influence of schedule delays, the number of transfers, crowding and reliability.
Kong (2013) used structural equation modelling to analyse the behavioural intentions in the context of time-of-day pricing of bus services in Chengdu, China. The findings of exploratory factor analysis highlighted six factors of behavioural intention and attitude, subjective norm, perceived behavioural control, acceptable fare range and socio-demographics. The estimated conceptual models indicated that commuter’s intention to departure time shift and travel mode shift was directly affected by attitudes and perceived behavioural control. Also, the latter factor was defined as an endogenous variable that was partly explained by subjective norm in the path analysis.

Lin and Feng (2013) measured the sense of loss aversion of different groups of metro travellers against the background of ticketing reform in Beijing, China. A macroscopic view considering the financial interests of traveller, transport agency and government subsidy was illustrated to assist in seeking an appropriate ticketing policy. The feasibility of duration-based, distance-based and zone-based ways was discussed thoroughly. The proposed methodology for analysing price elasticity is still helpful in addressing other pricing related issues.

In a summary of the above empirical studies, the validity of various differential pricing policies was assessed and then confirmed. Nevertheless, there are some common assumptions or limitations in previous work awaiting further improvements. The first thing is to distinguish between traditional mode choice behaviour and mode shift behaviour influenced by differential pricing. As can be seen, some studies employed the public’s intention towards metro pricing as a rough reference to support differential pricing policy-making. Inevitably, the habitual factors of regular metro commuters were observed restrictively and resulted in biases when evaluating policy effects. Thus, it is crucial to allow necessary ranges in departure time and mode alternatives in survey design and choice modelling. Only in this way, commuters’ responses to the differential policy can be in line with expectations in the real-life context and provide the necessary support for policy-making.

2.3 Methods for Developing Differential Pricing Strategies

Given the continuous surge of ridership in many metro systems, previous studies on static pricing issues cannot meet the needs of developing differential pricing strategies in practice anymore. Consequently, some related work on optimising appropriate strategies has been carried out.

In the early days, some scholars attempted to find possible ways to implement differential pricing strategies in metro systems. Although without giving specific mathematical methods, some valuable suggestions were put forward. Odlyzko (1999) suggested that differential pricing is a useful tool for travel demand management in terms of improving the service quality of Paris metro. Through a demand elasticity analysis against pricing factors, a practical framework of differential pricing was proposed, as the author believed that higher prices result in less ridership and bring about better service. With the purpose of enhancing metro’s competitiveness in the whole public transport system, Qin and Jia (2007) intended to maximise public benefits by substituting price elasticity into a standard Ramsey pricing function. In Particular, the revenue guarantee of metro company was considered in the form of a breakeven constraint.

Afterwards, the literature on developing pricing strategies mostly focused on Ramsey-based pricing models and other mathematical optimisation methods. Younes and Siriphong (2010) aimed to achieve the least overall delay in an equilibrium condition by adjusting metro fares. Notably, the metro was emphatically described as a schedule-based transport system that calls for a series of improvements on measuring the overall travel delay compared to the vehicular traffic system, which is the most innovative part of their work. Wang and Zhou (2012) introduced temporal weights to modify the Ramsey pricing model, which highlights the equity in public policy-making. Gong and Jin (2014) applied trilateral game theory to pricing modelling to seek the balance between government, transport agency and passenger. The most important contributions of the above research can be summarised as the precious guidance for pricing scheme adjustment needs during specific development phases, which made a breakthrough to tradition pricing problem. However, gaps in the differential pricing strategy design remain to persist.

In the past few years, the methodology for optimising differential pricing strategies is receiving growing concerns not only on metro systems but also has been applied to other transport systems. To start from the fields outside the metro, Sabounchi et al. (2014) and Zheng et al. (2016) proposed simulation methods for developing area-based dynamic pricing schemes for the congested road network. The former focused on road traffic’s interaction with public transport as drivers tend to choose an alternative mode in less demand and better service when extra payments to get access to specific areas are required. The latter presented an in-depth analysis of the
significant role of heterogeneity across drivers in incentivising them to shift to public transport. Wu et al. (2019) presented a two-stage optimisation model for dynamic pricing of high-speed rail. Based on the proposed seat allocation and pricing schemes, the ticket revenue of enterprise rose by 4.47% in the context of Beijing-Shanghai corridor. Tang and Lam (2019) engaged in the differential fare scheme design for limited-stop bus service to better release the potentials of bus transit. Three possible charging rules between origin and destination of time-based, stop-based and quality-based pricing were designed and then evaluated in terms of the total social cost.

Benefiting from existing works on other transport systems, further progress has been achieved regarding metro differential pricing. For example, Huang et al. (2016) presented an application of differential pricing named Paris Metro Pricing (PMP), seeking to motivate mobile subscribers to book different train classes as tickets information for the coming 24 hours is available online and in dynamic changes. Apart from qualitative types of methods, bi-level programming is widely adopted in addressing differential pricing issue. Peng et al. (2016) attempted to maximise the generalised benefits of all interested parties in the upper level optimisation model. Li et al. (2017) employed the cellular automation model to modify tradition heuristic algorithm for better performance in searching for the optimal solution of the proposed bi-level model. Intending to optimise the mode share pattern in the multimodal transport system, Liu and Wang (2017) modelled the metro differential pricing problem for minimising the total travel time.

This study proposes a bi-level optimisation model for developing differential pricing strategies to fill current gaps in combining choice modelling with conventional optimisation programming. Furthermore, insights on metro commuters’ departure time and mode shift behaviour, as well as effects evaluation of various decision-oriented strategies were presented.

3 Methodology

When a differential pricing strategy is scheduled to be implemented, the impacts of policy elements such as the affected periods and stations, the level of off-peak discount and extra peak charge need to be understood a priori to guarantee the intended effects. We thus conduct a stated preference (SP) survey regarding commuters’ responses to differential pricing strategies. Based on the estimated Nested Logit (NL) model, a bi-level optimisation model comprised of a multi-objective optimisation model in the upper level and a stochastic user equilibrium model in the lower level is further formulated, together with a solution algorithm to the models.

3.1 Modelling Departure Time and Mode Shift Joint Choice Behaviour

3.1.1 SP Survey Design and Implementation

The SP survey is designed to identify the potential factors that influence commuting behaviour in the context of differential pricing. Specifically, the questionnaire presents two sets of SP choice tasks concerning commuting behaviour in the morning and evening peaks. Figure 3 displays a set of SP choice tasks as the example, where this is developed based on a real commuting route between Huilongguan community to Xizhimen centre area in Beijing.

In the actual survey, each SP task shows ten alternatives of metro departures covering a wide range of departure times, and three alternatives involving shifts to other travel modes. We base the scenarios on a regular commuting plan, given that commuter’s previous travel habits might influence the intention of adjusting previous behaviour. It should be noticed that the regular departure time directly determines how difficult it would be for commuters to access the off-peak discount. For example, if a commuter usually leaves home at 7:10 am, and the off-peak discount is planned to ends by 7:00 am, then the reduced fare is available for this commuter subject to a shift to an earlier departure by a minimum of 10 minutes. By contrast, for those departing at 8:00 am, the off-peak discount would be less attractive as a significant change on the regular schedule is required. In this regard, it is essential to include the sense of relative adjustment in questionnaire design, which highlights the significance of finding the target group when developing a public transport policy. As for the forms of differential pricing, apart from the most commonly used off-peak discount, the extra peak charge is also taken into account for further strengthening the policy effects.
Example of hypothetical commuting SP tasks

Assuming that you live in Huilongguan community and work near Xizhi Men, the overall situations of commuting by metro Line B1 are summarized as below, along with two candidate differential pricing schemes that might be adopted in the near future.

<table>
<thead>
<tr>
<th>Scenario 1#</th>
<th>Plan No.</th>
<th>Travel by</th>
<th>Leave at</th>
<th>Arrive at</th>
<th>Duration</th>
<th>Crowdedness</th>
<th>Ticket Price (CNY)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Current</td>
</tr>
<tr>
<td>1</td>
<td>6:35</td>
<td>7:40</td>
<td>1h5min</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6:40</td>
<td>7:55</td>
<td>1h20min</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>6:50</td>
<td>8:10</td>
<td>1h20min</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>6:55</td>
<td>8:25</td>
<td>1h30min</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>7:10</td>
<td>8:40</td>
<td>1h30min</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>7:25</td>
<td>8:55</td>
<td>1h30min</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>7:50</td>
<td>9:10</td>
<td>1h20min</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>8:02</td>
<td>9:25</td>
<td>1h20min</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>8:35</td>
<td>9:40</td>
<td>1h5min</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>8:50</td>
<td>9:55</td>
<td>1h5min</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>11</td>
<td>Bus</td>
<td>Around 1h40min</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>Taxi</td>
<td>Around 1h10min except waiting time</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>71</td>
</tr>
<tr>
<td>13</td>
<td>Car</td>
<td>Around 1h10min</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>65</td>
</tr>
</tbody>
</table>

According to the above situations, if the commuting time arrangement that you got used to is as below, which plan would you like to choose when pricing scheme II is going to be put into practice?

<table>
<thead>
<tr>
<th>Regular plan</th>
<th>Please tick in the box below the plan you would like to choose now</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>3 – arrive at 8:10</td>
<td>0-15</td>
</tr>
<tr>
<td>4 – arrive at 8:25</td>
<td>0-25</td>
</tr>
<tr>
<td>5 – arrive at 8:40</td>
<td>0-35</td>
</tr>
<tr>
<td>6 – arrive at 8:55</td>
<td>0-50</td>
</tr>
<tr>
<td>9 – arrive at 9:40</td>
<td>0-120</td>
</tr>
</tbody>
</table>

Fig. 3 Example of a hypothetical scenario in the questionnaire

A face-to-face form of interviews was conducted in the areas within the Fifth Ring Road in Beijing during May 2018. Considering only regular metro commuters are qualified to respond, the interviews mostly took place in shopping centres around metro stations during after-work hours. There are three steps in an interview. The interviewee is asked about their commuting mode choice at the very beginning. Only qualified metro commuters move on to the second step, where a brief introduction is arranged to ensure the respondent has the basic knowledge of differential pricing. The interviewer then guides the respondent to complete the questions one by one and provides assistance where needed. In this way, a total of 2,467 valid samples from 135 respondents are collected. Figure 4 presents the descriptive statistics of the collected samples.

Fig. 4 Descriptive statistics of the collected samples
According to the preliminary statistics of socio-demographics, most of the indicators distribute evenly across respondents, although there is a high proportion of female interviewees. The reason known from interviewers’ feedback is that women express more willingness to accept the survey invitation than men. Even in those cases that a couple participated, women are more likely to be the one completing the questionnaire as a representative. Additionally, the statistical results of respondents’ responses to departing earlier are summarised in Figure 5.

![Fig. 5 Statistical results of SP choices towards departing earlier](image)

Scheme II is designed to allow for stronger incentivised effects compared to scheme I. The results are statistically in line with the expectation as scheme II attracts more commuters to depart earlier, especially evident in the higher proportions of accepting the 20-min and 35-min ahead options. Also, some primary hypotheses regarding differential pricing can be speculated from that. Generally, scheme I is capable enough if intending to attract commuters to make a 5-min adjustment. When the enhanced effects are desired, the extra peak charge in scheme II is preferred as 65% of respondents are willing to depart more than 20 minutes before. Figure 5 also indicates that a loss of metro ridership probably occurs when the differential pricing strategies take effects.

### 3.1.2 Nested Logit Model Specification

Based on random utility theory, the utility function for alternative $i$ and decision maker $n$ consists of the following two parts

$$ U_{ni} = V_{ni} + \varepsilon_{ni} $$

where $V_{ni}$ and $\varepsilon_{ni}$ are the deterministic and stochastic terms of the utility function, respectively.

Under the assumption of a type I extreme value distribution, we would obtain a logit model, with the probability of commuter $n$ choosing alternative $i$ out of $J = 1, 2, ..., J$ given by:

$$ P_{ni} = \frac{e^{V_{ni}}}{\sum_{j=1}^{J} e^{V_{nj}}} $$

where $0 \leq P_{nj} \leq 1$, $\forall j, n$ and $\sum_{j=1}^{J} P_{nj} = 1$, $\forall n$

Under the assumption of a generalised extreme value distribution, we instead use the nested logit (NL) structure, where alternatives are grouped into mutually exclusive nests. The probability of a commuter choosing alternative $i$, where $i \in m$, with $m$ being one of $R = 1, 2, ..., R$ different nests, is then given by:

$$ P_{nm} = P_{nim} \times P_{n} $$

where $P_{nim}$ is the conditional probability of alternative $i$ being chosen in the nest $m$; $P_{n}$ is the marginal probability of nest $m$ being chosen.

The equations of calculating $P_{nim}$ and $P_{n}$ can be expressed as

$$ P_{nim} = \frac{e^{V_{nim}}}{\sum_{j=1}^{J} e^{V_{nj}}} $$

$$ P_{n} = \frac{e^{V_{n}}}{\sum_{r=1}^{R} e^{V_{nr}}} $$
\[ V^*_m = \frac{1}{\mu_2} \ln \sum_{j=1}^{J} e^{\nu_j \beta_m} \]  

where \( V^*_m \) is the logsum term reflecting the impact of lower level on the upper level; \( \mu_1, \mu_2 \) are the scale parameters of the upper and lower level. As the normalisation is done at the bottom, \( \mu_1 = 1, \mu_2 < 1 \); \( V^*_m \) is the lower-level utility of choosing alternative \( i \) in the nest \( m \) perceived by the commuter; \( V^*_m \) is the upper-level utility of choosing nest \( m \) perceived by the commuter.

Intuitively, there might be differences in commuters’ preference for adjusting commuting behaviour in the morning and evening peak times. Commuters often have more choices for after-hour activities rather than going back home immediately, which probably leads to a difference in the value of time. We thus estimate the choice models for the morning and evening peak separately. The NL model structure is illustrated in Figure 5.

![Fig. 6 NL model structure](image)

The upper level shows whether commuters keep travelling by metro or shift to other travel modes like bus, taxi and private car. The lower level describes commuter’s departure time adjustment, including maintaining the regular schedule, departing earlier or later in various time ranges.

We have four nests referring to the nests of metro, bus, taxi and private car, respectively. We use \( A_i \) to denote the alternative in the first nest of metro in Table 1, where the attributes used in the utility function are listed.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Metro</th>
<th>Bus</th>
<th>Taxi</th>
<th>Private car</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Attributes</strong></td>
<td>( A_1 )</td>
<td>( A_2 )</td>
<td>( \ldots )</td>
<td>( A_j )</td>
</tr>
<tr>
<td>Upper level constant (ASC)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>Travel time</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>Travel cost</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>Lower level Shift in departure time</td>
<td>( \checkmark )</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Crowding degree</td>
<td>( \checkmark )</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

1 The tick mark indicates the attribute in the horizontal row is included in the utility function of the alternative in the vertical column. The short dash refers to the opposite meaning.

The entire sample set is divided into two subsets in terms of the peak types. We then use Guass 16.0 to calibrate the NL models with the classified SP data. The model estimation results are presented in Table 2.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Unit</th>
<th>Morning peak coefficient</th>
<th>Morning peak t-test</th>
<th>Evening peak coefficient</th>
<th>Evening peak t-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower-level scale parameter</td>
<td>-</td>
<td>0.967</td>
<td>3.140</td>
<td>0.5580</td>
<td>3.144Lowe</td>
</tr>
<tr>
<td>ASC for taxi</td>
<td>-</td>
<td>12.471</td>
<td>2.201</td>
<td>17.843</td>
<td>2.750</td>
</tr>
<tr>
<td>ASC for private car</td>
<td>-</td>
<td>10.564</td>
<td>2.046</td>
<td>15.546</td>
<td>2.632</td>
</tr>
<tr>
<td>Travel time</td>
<td>h</td>
<td>-8.068</td>
<td>-11.198</td>
<td>-5.651</td>
<td>-14.828</td>
</tr>
<tr>
<td>Travel cost</td>
<td>CNY$^1$</td>
<td>-0.250</td>
<td>-2.870</td>
<td>-0.317</td>
<td>-3.180</td>
</tr>
<tr>
<td>Shift in departure time</td>
<td>h</td>
<td>-5.552</td>
<td>-3.238</td>
<td>-2.446</td>
<td>-3.204</td>
</tr>
<tr>
<td>Crowding degree$^2$</td>
<td>%</td>
<td>-0.333</td>
<td>-2.832</td>
<td>-0.206</td>
<td>-2.853</td>
</tr>
<tr>
<td>Sample size</td>
<td>1245</td>
<td>1222</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted ( \rho^2 )</td>
<td>0.417</td>
<td>0.388</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 CNY/USD \( \approx 0.145 \) during the survey period. 2 Crowding degree is quantified by the average section load rate

All the estimated coefficients of situational attributes presented in Table 2 have the absolute t-value greater
than 1.96, indicating the significant influences on commuting behaviour. The variables of travel time, travel cost, shift in departure time and crowding degree show significant negative impacts on departure time and mode shift behaviour as expected. The adjusted $\rho^2$ of the two models are 0.417 and 0.388, which perform well in terms of model fit. Also, the value of time (VOT) reflected by time-related attributes are further calculated based on Equation (7). The VOT estimation results are presented in Table 3.

$$\mu = \beta_t / \beta_v$$  \hspace{1cm} (7)

where $\mu$ is VOT of a time-related variable, CNY/h; $\beta_t$, $\beta_v$ are the estimated coefficients of a certain time-related variable and travel cost.

<table>
<thead>
<tr>
<th>Time-related variable</th>
<th>Morning peak</th>
<th>Evening peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel time</td>
<td>32.3 CNY/h</td>
<td>17.8 CNY/h</td>
</tr>
<tr>
<td>Shift in departure time</td>
<td>22.2 CNY/h</td>
<td>7.7 CNY/h</td>
</tr>
</tbody>
</table>

According to the data released by Beijing Municipal Bureau of Statistics, the per capita disposable income of Beijing is 62,361 CNY in 2018. For a reference to the obtained VOTs, the mean working VOT of Beijing residents is estimated at 29.52 CNY/hour as there are 8 working hours a day, 22 working days a month and a total of 2,112 working hours a year.

The VOTs in the morning peak are higher than those in the evening peak as the morning peak is always more crowded and less comfortable. We also speculate that commuters usually enjoy more flexibility in adjusting the after-work schedule. In most cases, commuter’s primary task in the morning is to avoid being late for work. However, for the evening peak, a growing number of people tend to spend more time on after-work activities, which also helps to avoid suffering transit crowding and higher fare in peak times. In this regard, differential pricing strategies usually perform better in the evening peak times and are less likely to lead to a loss of ridership.

We also notice that the sensitivity to shifting departure time is less strong than that for travel time. This is closely related to the natural differences between these two attributes because travel time is the real consumption of time that commuters have to afford to complete the daily round-trip. However, when commuters reschedule the commuting plan to access a reduced fare, this is a scheduling disutility, but the time can still be used for other activities.

### 3.2 Bi-level Programming Formulation

Based on the estimated choice model, this section presents the bi-level programming of developing differential pricing strategies. The upper level optimisation model intends to balance the interests of metro agency, commuters and operating risks by optimising the multi-objective of maximising ticket revenue, minimising travel cost and maximum ridership in peak times. The lower level specifies a NL-based SUE model to accommodate variations of travel demand under the influence of differential pricing strategy received from the upper level model. With the demand pattern feedback from the lower level, the upper level seeks the optimal solution continuously.

#### 3.2.1 Upper level programming of differential pricing optimisation model

The upper level model aims to ensure a proper optimisation direction to avoid a local optimum solution. Metro agencies intend to relieve the crowding during peak hours and alleviate the accompanying risks while at the same time maintaining or even improving current profits of ticket sales. On the other hand, commuters have a preference for better service and lower prices. In order to create a win-win situation, the multi-objective programming is used at the upper level to include objectives on behalf of different interest groups.

We consider the set of research periods $K$, $k \in K$, where the subset of $K$, $K_1$ and $K_2$ represents the sets of peak and off-peak periods. Also, we define the set of O-D (Origin-Destination) pairs as $W$, $w \in W$. The ticket revenue of metro agency can be measured by

$$E = \sum_{w \in W} \sum_{k \in K} p_w q^*_{w} (k) \delta_w (k), \hspace{1cm} r = 1$$  \hspace{1cm} (8)

where $E$ is the amount of ticket revenue, CNY; $p_w$ is the base fare between O-D pair $w$; $\delta_w (k)$ is the
differential fare rate between O-D pair \( w \) during period \( k \); \( q^w_{mk} \) is the number of commuters travelling by mode \( r \) between O-D pair \( w \) during period \( k \).

The travel cost of commuters includes both time and monetary cost. With the aid of the VOT coefficient, the generalised time cost can be written as

\[
C_T = \mu \sum_{w \in W} \sum_{k \in K} \sum_{r \in R} q^w_{mk} t^r(k)
\]

(9)

where \( C_T \) is the generalised time cost of commuters, CNY; \( t^r(k) \) is the travel time between O-D pair \( w \) travelling by mode \( r \) during period \( k \), h; \( \mu \) is the VOT coefficient, CNY/h.

Considering the metro ridership varies with the changes of differential pricing strategies, the total monetary cost ought to cover commuters either still taking metro or shifting to other modes, which can be expressed as

\[
C_v = \sum_{w \in W} \sum_{k \in K} \sum_{r \in R} q^w_{mk} p^r_{mk}(k)
\]

(10)

\[
p^r_{mk}(k) = p^r_{mk}(k), \ r = 1
\]

(11)

where \( C_v \) is the monetary cost of commuters, CNY; \( p^r_{mk} \) is the ticket price of mode \( r \) between O-D pair \( w \) during period \( k \), CNY. Note that except \( r = 1 \), \( p^r_{mk} \) relates solely to the O-D pair \( w \) and does not vary with period \( k \).

Based on the above specifications, the upper level optimisation model is formulated, such that:

\[
\text{max } Z_1 = \text{max } \sum_{w \in W} \sum_{k \in K} \sum_{r \in R} p^r_{mk}(k) \delta^r_{mk}(k), \ r = 1
\]

(12)

\[
\text{min } Z_2 = \min \left( C_T + C_v \right) = \min \left[ \mu \sum_{w \in W} \sum_{k \in K} \sum_{r \in R} q^w_{mk} t^r(k) + \sum_{w \in W} \sum_{k \in K} \sum_{r \in R} q^w_{mk} p^r_{mk}(k) \right]
\]

(13)

\[
\text{min } Z_3 = \max \left[ q^w_{mk}(k) \right], \ r = 1
\]

(14)

subject to

\[
\delta^w_{mk}(k) \leq 1, \ \forall w, k \in K;
\]

(15)

\[
1 \leq \delta^w_{mk}(k) \leq \delta^w_{max}, \ \forall w, k \in K;
\]

(16)

\[
q^w_{mk}(k) \geq 0, \ \forall w, k \in K, r \in R;
\]

(17)

where \( Z_1, Z_2, Z_3 \) are the three objectives of the optimisation model; \( \delta^w_{min}, \delta^w_{max} \) are the lower and upper thresholds of differential fare rates.

In Equation (15) and (16), the upper bound of differential fare rate is set to ensure necessary social welfare. Oppositely, the lower bound is also essential to guarantee the interests of metro agency. Also, the ridership of any travel mode must meet the non-negative value constraint as Equation (17).

### 3.2.2 Lower level programming of stochastic user equilibrium model

The commuting travel demand in the metro system is usually characterised as the distribution pattern of passenger flow during peak hours. The flow pattern is one of the most important references for developing differential pricing strategies. Oppositely, the implementation of demand management policy results in changes in the flow pattern. It is thus crucial to create a feedback loop between the flow pattern and policy intervention. Instead of merely assuming all commuters follow the least-cost choice as in the deterministic user equilibrium model, stochastic user equilibrium (SUE) models characterise the individual’s decision-making process as dynamic. As a consequence, each alternative in the choice model has a non-zero probability of being (Zhang et al. 2019). The SUE model defines a situation where no user can improve the perceived travel cost by changing alternatives unilaterally (Prashker and Bekhor 2004).

There are two SUE conditions derived from each level of the NL model. The first layer is the equilibrium situation of mode choice and the second layer pursues the equilibria of the generalised cost among departure time alternatives in the nest of metro. The generalised cost of alternatives in the NL model can be written as
\[ C^n_r(k) = -V^n_r(k), \quad r = 1 \]
\[ C^n_r(k) = \left\{-\frac{1}{\lambda_2} \ln \sum_{r} \exp[\lambda_2 V^n_r(k)], \quad r = 1 \right\} \]
\[ -V^n_r(k), \quad r \neq 1 \]

where \( C^n_r(k) \) is the generalised cost of choosing mode \( r \) during period \( k \); \( C^n_{ja}(k) \) is the generalised cost of choosing alternative \( j \) in the nest \( r \) during period \( k \).

We then define the minimum expected cost of choosing mode \( r \) as \( E\left[ \min \left[ C^n_r(k) \right] \right] \), with commuters’ perceived cost of choosing mode \( r \) given by:
\[ S\left[ C^n_r(k) \right] = E\left[ \min \left[ C^n_r(k) \right] \right] = -\frac{1}{\lambda_2} \ln \sum_{r} \exp[\lambda_2 C^n_r(k)] \]

In the SUE condition, the travel mode chosen by any user is with the same minimum cost, applying to each O-D pair \( w \) and period \( k \), such that:
\[ \sum_{r} q^n_{rw}(k) \left[ C^n_r(k) - S\left[ C^n_r(k) \right] \right] = 0 \] \( \quad (21) \)
\[ C^n_r(k) \geq S\left[ C^n_r(k) \right] \] \( \quad (22) \)
\[ q^n_{rw}(k) \geq 0, \quad \forall w \in W, \forall k \in K, \forall r \in R \] \( \quad (23) \)

For the lower level, we use \( S\left[ C^n_{ra}(k) \right] \) to denote commuters’ perceived cost of choosing alternative \( j \). The minimum expected cost of choosing alternative \( j \) is \( E\left[ \min \left[ C^n_{ra}(k) \right] \right] \). The SUE condition of departure time choice is given by
\[ S\left[ C^n_{ra}(k) \right] = E\left[ \min \left[ C^n_{ra}(k) \right] \right] = -\frac{1}{\lambda_2} \ln \sum_{r} \exp[\lambda_2 C^n_{ra}(k)], \quad r = 1 \]
\[ \sum_{w} q^n_{rw}(k) \left[ C^n_{ra}(k) - S\left[ C^n_{ra}(k) \right] \right] = 0, \quad \forall w \in W \]
\[ C^n_{ra}(k) \geq S\left[ C^n_{ra}(k) \right] \] \( \quad (24) \)
\[ q^n_{rw}(k) \geq 0, \quad \forall w \in W, \forall k \in K, \forall j \in J \]

where \( q^n_{rw}(k) \) is the number of commuters choosing alternative \( j \) in the nest \( r \) between O-D pair \( w \) during period \( k \).

Furthermore, the integrated NL-SUE model can be written equivalently as the following variational inequality formulation.
\[ \sum_{w} q^n_{rw}(k) \left[ q^n_{rw}(k) - q^n_{rw}(k) \right] \left[ C^n_r(k) + \frac{1}{\lambda_2} \left[ \ln(q^n_{rw}(k)) - \ln(q^n_{rw}(k)) \right] - S\left[ C^n_r(k) \right] \right] \geq 0 \] \( \quad (25) \)
\[ \sum_{w} q^n_{rw}(k) \left[ q^n_{rw}(k) - q^n_{rw}(k) \right] \left[ C^n_r(k) + \frac{1}{\lambda_2} \left[ \ln(q^n_{rw}(k)) - \ln(q^n_{rw}(k)) \right] - S\left[ C^n_r(k) \right] \right] \geq 0 \] \( \quad (26) \)

In Equation (28), the optimal solutions of \( u^n_r(k) \) and \( u^n_{ra}(k) \) exist in the feasible region subject to Equation (3-6), (18-19), (23) and (27). Also, the strict convexity is demonstrated by the positive definiteness of Hessian matrix, indicating the uniqueness of the SUE solution. Through analysing the Karush-Kuhn-Tucker (KKT) conditions of the lower level model, the equivalence between the proposed SUE conditions and given variational inequality formulation is further confirmed.

### 3.3 Solution Algorithm

The bi-level programming is the NP-hard problem that has high calculation complexity of solving. Previous research suggests that heuristic algorithms such as genetic algorithm, simulated annealing algorithm, particle swarm algorithm perform better than the numerical algorithms in light of calculation efficiency and the quality of optimal solution (Labbé and Violin 2016; Hemmati and Smith 2016). Also, considering the outstanding
performance of MSWA (Method of Successive Weight Average) on reaching a better convergence in the SUE problem, a genetic algorithm-based MSWA solution procedure is developed to solve the proposed bi-level model.

3.3.1 Computing the optimal differential pricing strategy

Given the three optimisation objectives in the upper level, a normalised way is used to standardise the dimensions of objectives. The extreme values of each objective are estimated in advance by solving each single-objective optimisation problem. We then simplify the multi-objective optimisation model using the fuzzy-compromise decision-making method. The modified single-objective function is given by

$$\max Z = \max \alpha_u u_t + \alpha_e (u_t - u_s)$$

\[ u_s = \frac{Z_{\text{max}} - Z_e}{Z_{\text{max}} - Z_{\text{min}}} \]

where \(u_t, \alpha_e\) are the allocation and weight coefficients of objective \(Z_e, e \in \{1, 2, 3\}\), representing maximising ticket revenue, minimising travel cost and maximum ridership, subject to \(\sum \alpha_e = 1\); \(Z_{\text{max}}, Z_{\text{min}}\) are the maximum and minimum values of objective \(Z_e\).

We set the differential fare rate \(\delta\) as the gene in the genetic algorithm, which plays the role of decision variable in the optimisation process. The chromosome is made up by a set of genes, forming the individual in the population. The best individual carries the optimal solution of differential fare rates when convergence is reached. Also, several essential procedures like selection, mutation and crossover are comprehensively included in the genetic algorithm. Specifically, we use the gambling-wheel disk selection method to support individual selection. The two-point crossover mode allows for adequate changes in the existing chromosomes. Besides, a mutation design is introduced to prevent incorrect convergence on a local optimum solution. The complete algorithm works as the following steps.

Step 1: Parameters initialisation. The related parameters include binary codes of genes, population size, the probability of mutation and the maximum iterations. The first generation of population is initialised subject to the constraints set of the upper level optimisation model.

Step 2: Fitness assessment. The currently best individual is substituted into the lower level model to obtain the SUE solution. Consequently, the fitness of the individual can be measured by the objective function.

Step 3: Retaining better individuals. All the individuals in the current population are prioritised in terms of the fitness performance. The better individuals are then delivered to the next generation with higher probabilities based on the gambling-wheel disk selection.

Step 4: Population evolution and convergence judgement. The two-point crossover and stochastic mutation are applied to the generating of the next population. Keep repeating the above steps and end the loop until a specified condition of convergence is reached.

3.3.2 Computing the NL-SUE solution

In the lower level model, the main goal is to measure the departure time and mode shift choice patterns under the influence of differential pricing strategy generated by the upper level model. The MSWA-based method is used to solve the SUE conditions, including the following four steps.

Step 1: Parameters initialisation. Set initial values for essential parameters. Let the number of iterations \(g = 1\), the lower error bound \(\varphi = 10^{-4}\), the initial period \(k = 0\), the flow rates of travel mode \(q_{fr}^e, k = 0\), the flow rates of departure time \(q_{fr,w}^e, k = 0\), the additional flow rates \(\bar{q}_{fr,w}^e, k = 0\) and \(\bar{q}_{fr,w}^{e+1}, k = 0\).

Step 2: Update the utilities of alternatives. Based on the current status of situational variables, compute the utilities of alternatives \(V^{(e)}_{r,w}(k)\), \(V^{(e)}_{fr,w}(k)\). In the meanwhile, the probabilities of choosing each alternative \(P^{(e)}_{r,w}(k), P^{(e)}_{fr,w}(k)\) are updated synchronously.

Step 3: Flow assignment. Take the departure time choice as an example, the flow rates in the current iteration \(q_{fr,w}^e, k\) can be written as

\[ q_{fr,w}^e, k = q_{fr,w}^{e+1}, k + \frac{1}{g}[\bar{q}_{fr,w}^{e+1}, k + q_{fr,w}^{e+1}, k] \]

(31)
\[
q_{j,p,w}^k = q_{j,p,w}^{k-1} \cdot p_{j,p,w}^{k-1} + 1
\] (32)

Note that the values of situational variables keep being updated at the end of this step.

Step 4: Convergence judgement. Move back to step 2 if there is a next period in the loop. If not, further judge whether the convergence condition is reached by Equation (33). If failing to end the convergence, continue iterating.

\[
\sum \sum \sum \left| q_{j,p,w}^{(e)}(k) - q_{j,p,w}^{(e-1)}(k) \right| < \epsilon
\] (33)

By combining the above two parts of solution procedures, the proposed bi-level optimisation model can be solved effectively. Figure 7 displays the flowchart of the solution algorithm for an explicit illustration.

Fig. 7 Flowchart of the solution algorithm

At the beginning of the algorithm, the initialised solution is created as the input of the lower level model. Subsequently, the SUE conditions can be obtained, and the current solution will be assessed by the fitness function after being forwarded back to the upper level model. As such, a new solution will be created and passed to the lower model again. All the possible solutions get through the above process over and over again until a qualified optimal solution is found.
4 Case study

4.1 Regional context

In this case study, we choose the Batong (BT) Line as the research object to examine the proposed bi-level optimisation model. BT Line connects the eastern part to the centre area of Beijing and currently has 13 stations in operation, as shown in Figure 8.

The right end of BT Line is located in a residential area of East Beijing, where a massive number of commuters take metro to the central business district every morning. The BT Line plays a crucial role in supporting the West-East commuting corridor of Beijing. With the continuous growth of metro ridership in recent years, serious flow control measures are currently implemented to mitigate the congestion during the morning peak. Although these steps work in the short term, the impacts on service quality are not ideal. In this context, we expect the differential pricing to contribute to reshaping the commuting patterns in an incentivised manner.

For the sake of simplicity, a single O-D pair is used to represent the typical group of commuters as the blue dotted lines marked in Figure 8. The origin area is composed of Liyuan (LY), Linheli (LHL) and Tuqiao (TQ) stations in the residential area. The destination area covers five busy stations in the centre area, which are Yonganli (YAL), Guomao (GM), Dawanglu (DWL), Sihui (SH) and Sihuidong (SHD). Since the morning peak in the research area is the biggest bottleneck in daily operation, the period of study is set between 6:00 am and 9:00 am, where the peak hour is between 7:00 am and 8:00 am. The periods either side are defined as the off-peak hours in the weekday morning.

Data on travel demand in the study period is extracted from tapping in and tapping out records collected by the AFC system. Also, the current pricing scheme, train capacity and timetable are gathered for the use in the upper level optimisation model. In particular, access to the situational attributes of the substituted travel modes was obtained with Baidu Web Application Programming Interface (API) – a professional tool for developers to use real-time navigation data provided by Baidu Map. All the attributes involved in the utility functions are thus well-prepared to support the solving of the lower level SUE model.

4.2 Sensitivity analysis of the pricing factor

As known from the NL model estimation results, the pricing factor influences commuter’s choice negatively, while other factors, such as crowding and travel time, also matter. To evaluate the pricing sensitivity, ten typical pricing schemes are proposed to allow for policy effects from weak to strong, as presented in Table 4.

<table>
<thead>
<tr>
<th>Scheme No.</th>
<th>Description</th>
<th>Differential fare rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>off-peak</td>
</tr>
<tr>
<td>I</td>
<td>Current scheme</td>
<td>1</td>
</tr>
<tr>
<td>II</td>
<td>Off-peak discount</td>
<td>0.75 1</td>
</tr>
<tr>
<td>III</td>
<td>Extra peak charge</td>
<td>0.5</td>
</tr>
<tr>
<td>IV</td>
<td>Both-sides way</td>
<td>0.25</td>
</tr>
<tr>
<td>V</td>
<td>1</td>
<td>1.25 2</td>
</tr>
<tr>
<td>VI</td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>VII</td>
<td>1</td>
<td>1.75</td>
</tr>
<tr>
<td>VIII</td>
<td>0.75</td>
<td>1.25</td>
</tr>
<tr>
<td>IX</td>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>X</td>
<td>0.25</td>
<td>1.75</td>
</tr>
</tbody>
</table>
In this example, the value of fare rate during off-peak hours is less than 1, indicating a 25% off on the base fare. By contrast, the fare rate higher than 1 refers to a 25% extra charge on the base fare.

There are usually three ways for implementing a differential pricing strategy, offering lower fares during off-peak periods as in scheme 2, 3 and 4, charging higher prices during peak periods as in scheme 5, 6 and 7, and adopting both of the above as in scheme 8, 9 and 10. For practical concerns, fare rates herein are all set as the integral multiples of 5% to guarantee necessary feasibility in practice.

Based on the schemes in Table 4, the SUE solutions can be worked out by the algorithm proposed above. By accumulating the number of commuters departing in each 15-min period, the whole patterns of metro ridership are shown in Figure 9.

![Fig. 9 Metro ridership patterns under typical differential pricing schemes](image)

With the rise of fare rates, considerable variations appear at the peak around 8:00 am. Since the off-peak discount is due at 7:00 am, commuters who used to depart between 7:00 am - 7:30 am have the highest probability of being incentivised to shift to the period of 6:45 am - 7:00 am. The ridership reduction during the pre-peak periods is therefore particularly significant. Due to the boundary point, the pricing factor does not take effects on cutting down the peak directly. Even so, the performance of moderating intensive arrival of commuters, hereafter called flow-regulation, is successfully verified as commuters reacting sensitively to those hypothetical pricing schemes. Also, it is essential to notice the variations of mode share patterns meanwhile. Figure 10 displays the updated mode share rates under the influence of differential pricing schemes.

![Fig. 10 Mode share patterns under typical differential pricing schemes](image)
Metro holds the overwhelming majority in any circumstance because the aggregate travel demand is sourced from the metro AFC data. The average loss of metro ridership is about 5%, with the least ridership-loss in scheme II and the highest ridership-loss in scheme VII. Notably, being different from the flow-regulation effects observed in Figure 9, the most potent flow-regulation scheme X appears to lose less ridership than many other schemes. The main reason for that is the weaker sense of paying more when both off-peak discount and extra peak charge are adopted. The commuters are therefore more willing to depart earlier rather than shifting to other travel modes when the extra peak charge exists. Namely, the extra peak charge counteracts the unwillingness of changing commuting habits, especially applied together with the off-peak discount. Furthermore, from an overall evaluation perspective, the estimated ticket revenue, generalised travel cost of the whole commuters and the maximum ridership within peak periods are contrasted in Figure 11.

![Fig. 11 Overall performance of typical differential pricing schemes](image)

The features of each type of pricing schemes stand out clearly in Figure 11. The three circles in light blue refer to schemes with off-peak discount only, bringing hardly any reduction on the peak ridership compared to the current pricing scheme represented by the light yellow circle. Those three schemes mostly benefit commuters while lowering the ticket revenue of the metro agency. In this regard, the off-peak discount scheme applies to the controllable scale of ridership and is highly recommended if there are adequate financial subsidies from the government.

As for the second set of schemes, V, VI and VII, we observe a similar spatial distribution with the third set of schemes VIII, IX and X. The comparable performances enable them to be analysed together. In terms of the maximum ridership, the implied flow-regulation effects tend to be stronger as the fare rate differences widen. The ticket revenue performs consistently with the fare rate setting in Table 4. Even though both-sides schemes provide the off-peak discount, the total revenue is slightly influenced because the ticket revenue during off-peak hours is not comparable with that during peak hours. In addition to the above two indicators, the most significant difference between these two sets of schemes lies in the travel cost of commuters. The second set of schemes perform worse in terms of the travel cost as no discount is offered for early birds.

Generally speaking, schemes with extra peak charge appear to be more effective than the off-peak discount schemes in the aspects of flow-regulation and ticket revenue retention even if the former puts extra burden on the travel cost of commuters. The off-peak discount is more appropriate to work as supplementary for the extra peak charge for two simple reasons. For one thing, the implementation of the off-peak discount will not reduce the current ticket revenue severely. From another point of view, the accompanying service improvement perceived by the commuter is rather considerable and proves to be beneficial for holding back the loss of ridership caused by the extra peak charge.

### 4.3 Analysis of optimal differential pricing strategies

Based on the pros and cons of different types of schemes discussed above, further steps are taken to seek objective-oriented optimal strategies with the proposed bi-level model. When it comes to a real-world application, the
differential pricing strategies take responsibility for all interest groups of metro agency, commuters and the government. In the general case, a balanced strategy is preferred, but undoubtedly, the optimal strategy varies with the priorities of policy-making. With different preferences for decision-making, several optimal strategies are further proposed for comparative analysis.

As mentioned earlier, \( \alpha \) represents the weight coefficient of optimisation objective, also implying the potential preferences in decision-making. With a series of combinations of weight coefficients, six optimal solutions of differential pricing schemes are obtained, as described in Table 5.

<table>
<thead>
<tr>
<th>Scheme No.</th>
<th>Description</th>
<th>Weight coefficient ( \alpha_1, \alpha_2, \alpha_3 )</th>
<th>Optimal fare rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Current scheme</td>
<td>[0.5, 0.25, 0.25]</td>
<td>1</td>
</tr>
<tr>
<td>II</td>
<td>Ticket revenue preferred</td>
<td>0.5, 0.25, 0.25</td>
<td>1</td>
</tr>
<tr>
<td>III</td>
<td>Commuter benefit preferred</td>
<td>0.25, 0.50, 0.25</td>
<td>0</td>
</tr>
<tr>
<td>IV</td>
<td>Operational safety preferred</td>
<td>0.25, 0.25, 0.50</td>
<td>0</td>
</tr>
<tr>
<td>V</td>
<td>Off-peak discount</td>
<td>0.40, 0.50, 0.10</td>
<td>0.3</td>
</tr>
<tr>
<td>VI</td>
<td>Extra peak charge</td>
<td>0.45, 0.45, 0.10</td>
<td>1</td>
</tr>
<tr>
<td>VII</td>
<td>Both-sides way</td>
<td>0.41, 0.45, 0.14</td>
<td>0.8</td>
</tr>
</tbody>
</table>

A total of seven schemes as presented. Among them, scheme I is the current pricing scheme as a reference. Scheme II, III and IV are aimed at strengthening one of the three objectives and place lower importance on the other two objectives. The other three schemes are categorised with the forms of differential pricing, having options of off-peak discount, extra peak charge and both-sides way as discussed in the sensitivity analysis.

As can be seen from the values of optimal fare rates, overemphasis on any single objective leads to the extreme results of differential fare rates. For instance, the ticket revenue preferred strategy pushes the rate of extra peak charge to the upper limit and does not allow any off-peak discount at all. Similarly, the commuter benefit preferred strategy intends to maximise off-peak discount, leaving the extra peak charge rate at the minimum. Seemingly, the given combinations of weight coefficients in scheme II, III and IV are unreasonable, which also demonstrates the superiority of multi-objective programming over a single-objective setting in solving the differential pricing optimisation problem.

Beyond that, we allow freely estimated weight coefficients in the optimisation processes of scheme V, VI and VII. The values presented in Table 5 are the optimal results obtained from an additional traversal embedded in the bi-level model. The finally applied weight coefficients of ticket revenue \( \alpha_1 \) and travel cost \( \alpha_2 \) are between 0.4 - 0.5, almost achieving a balance between the interests of the metro agency and commuters. Due to the unit difference between the third objective and the first two objectives, the optimal value of the third weight coefficient \( \alpha_3 \) is relatively lower at around 0.1, which helps to achieve a better balance among the three objectives.

The optimal fare rates results indicate the off-peak discount of up to 70% and the extra peak charge performing best at 60%. To further evaluate the above optimal strategies, the metro ridership patterns are shown in Figure 12.

![Fig. 12 Metro ridership patterns under optimal differential pricing schemes](image-url)
As presented in Figure 12, the operation safety preferred scheme IV reduces the peak ridership most significantly. However, the flow-regulation effects of scheme IV appear to be excessive as a massive number of commuters choose to depart during the pre-peak periods, which therefore leads to a new ridership peak. Except for the single-objective oriented strategies, scheme VI and VII perform moderately in mitigating the crowding during the peak periods, which are more advisable than scheme V. Furthermore, the mode share patterns under optimal differential pricing schemes are presented in Figure 13.

Given the metro share rates in Figure 13, scheme II results in the maximum loss of metro ridership of nearly 10%. Among scheme V, VI and VII, the first one lost less than 4% of ridership while the other two schemes raise the proportion to over 6%. Acting consistently with the weak performance of flow-regulation in Figure 12, scheme V changes the entire mode share pattern slightly. Also, commuters’ most favourite alternative travel mode is confirmed to be the bus, taking around 75% of the total amount of shifted commuters. In the end, the overall performance of the three optimisation objectives is visually displayed in Figure 14.

By observing the vertical axis of the maximum ridership, scheme II and IV work the best in relieving the peak crowding pressure. As pointed out earlier, these two schemes adopt extreme fare rates and lead to unsatisfactory overall performance, leaving scheme VI and VII as candidates for further consideration. Since scheme VI and VII achieve comparable performance in cutting down the peak of ridership, scheme VII can be deemed as an improved scheme of VI that partly shares benefits to commuters by offering the off-peak discount. In this regard, scheme VII is more recommended when the metro agency prefers the customer-first mindset or has access to subsidies or other forms of government assistance. As for the strategy of off-peak discount only, scheme IV falls short of expectations as it proved to not be attractive enough to incentivise commuters to make a change.
All the above results come from the empirical study in the context of the BT line in Beijing, and the representativeness is thus not guaranteed. There is still a possibility for the off-peak discount to be effectively adopted in a commuting corridor that has relatively competitive travel mode alternatives.

Conclusions

The present work proposed a thorough framework for developing differential pricing strategies. An empirical study in Beijing further demonstrates the effectiveness of the proposed methodology by sensitivity analysis of pricing factors. Also, six tailored strategies are set out for evaluating the adaptability of differential pricing in terms of metro ridership, mode share patterns, ticket revenue and the generalised travel cost. The research finding enables policymakers to develop an appropriate differential pricing strategy depending on their situations.

At first, some qualitative conclusions can be drawn from the sensitivity analysis results. (1) The off-peak discount is beneficial to commuters but powerless to reshape demand patterns. (2) The extra peak charge performs effectively for both flow-regulation and ticket revenue retention. From the application point of view, this type of strategy is usually hard to be widely implemented due to public opposition. Nevertheless, if metro overwhelmingly dominates other modes in a specific commuting corridor, the extra peak charge can be a sensible choice to incentivise travel demand and maintain company income. (3) By contrast, the both-sides strategy inherits the advantages of the above two types of strategies, in particular, off-peak discount reduces commuters’ reluctance to adjust previous travel habits compared to only adopting the extra peak charge. The accompanying loss of ticket revenue is acceptable owing to the lower reduction in ridership, which compensates the ticket revenue at the same time to some extent. Except for the optimisation complexity and implementation issues, no significant shortcomings were found in the both-sides strategy.

Based on the exposed pros and cons of different types of strategies known from the sensitivity analysis, six optimal strategies are put forward for further evaluation. The set of single-objective oriented strategies include the ticket revenue preferred, commuter benefit preferred and operational safety preferred schemes, which have unsatisfactory performance as optimising the single objective to the limit. We thus manage to incorporate the optimisation of weight coefficients into the bi-level model to seek better solutions for off-peak discount, extra peak charge and both-sides strategies. Based on the performance evaluation, the both-sides strategy is more appropriate in the context of the research area. The optimal fare rate of off-peak discount is up to 70% and the extra peak charge works best at 60%. This study not only provides insights into commuters’ behaviour but also enables metro agency to manage travel demand scientifically.

There are still limitations awaiting further improvements in future works. The first concern is associated with implementing differential pricing in the real-world context. With the fixed boundary point currently adopted in the optimisation process, commuters who depart much later than the given boundary point are less likely to be affected by the pricing policy. The rapid adoption of e-ticket creates opportunities to track individuals’ commuting activity in the foreseeable future. By that time, there will be a possibility to provide a customised reward plan for individuals who have the intention of adjusting personal commuting behaviour in response to the call for peak avoidance. The second limitation is the lack of information about passengers from other modes or those not currently travelling, who might be attracted by the differential pricing strategy. As specified in the NL model structure, metro commuter’s mode shift behaviour is considered, which is further verified to be less than 10% in the case study. Oppositely, we assume that few commuters will shift from other modes to metro and the induced demand is thus negligible. However, the evolution of mode share patterns in a commuting corridor deserves further research, which requires a broader range of investigation not only limited to metro commuters. The last limitation relates to traveler characteristics, both measurable (e.g. socio-demographic) and unmeasurable (e.g. attitudes). Such characteristics likely influence travel behaviour but these are not recorded in the AFC data and can thus not be used. In pursuit of better policy effects, future work in choice modelling is expected to improve the understanding of commuting behaviour using different data sources (e.g. revealed preference data obtained from the on-board WIFI connection records or e-ticket usage history). Also, the implementing form of differential pricing needs to be further explored, and more efforts should be devoted to the policy design of personalising peak-avoidance awarding in future works.

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Acknowledgement

This work was supported by the National Key R&D Program of China (No. 2018YFB1601300) and the Beijing Natural Science Foundation (No. 8171003). Stephane Hess acknowledges the support of the European Research Council through the consolidator grant 615596-DECISIONS.

Reference


Kong, P: Study on the intention of residents’ departure time choice under the conditions of the time-of-day pricing of bus[D]. Southwest Jiaotong University (2011)


Song, Q.: Research on subway fare related issues based on time difference pricing[D]. Beijing Jiaotong University (2011)


Yang, J., Jin, J., Wu, J., Jiang, X.: Optimising passenger flow control and bus-bridging service for commuting
metro lines. Computer-Aided Civil and Infrastructure Engineering, 32(6), 458-473 (2017)

